OCR
Oxford Cambridge and RSA

Practice Paper – Set 4

A Level Mathematics A

H240/01 Pure Mathematics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

FINAL

This document consists of 14 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

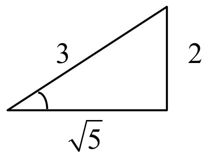
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			**DR** A: $C = 2m + 10$ B: $C = 2.4m + 3$ Where C is the total charge in pounds and m is the length of the journey in miles. $2.4m + 3 = 2m + 10$ $0.4m = 7$ $m = 17.5$ Same cost for a journey of 17.5 miles	 **M1** **A1** **M1** **A1** **[4]**	 **3.1b** **3.1b** **1.1** **3.2a**	 Either correct equation Both equations correct and variables defined. Attempt to solve simultaneously Obtain 17.5 miles	 Attempt to solve their two equations Units needed
2	**(a)**	**(i)**	$2 + 5 + x + x + (x + 2) + (x + 5) < 44$ oe	**B1** **[1]**	**1.1**	Correct inequality	Must be $<$ only
		(ii)	$x(x + 2) + 10 \geq 45$ oe	**B1** **[1]**	**1.1**	Correct inequality relating to area	Must be $\geq$ only
	(b)		$x < 7.5$ $x^2 + 2x - 35 \geq 0$ critical values are -7 and 5 $x \leq -7, x \geq 5$ but x is a length so $x \geq 5$ $\{x: 5 \leq x < 7.5\}$ or $[5, 7.5)$	**B1FT** **M1** **A1FT** **B1** **[4]**	**1.1** **1.1a** **2.4** **2.2a**	Obtain $x < 7.5$ from linear inequality FT their linear inequality in **(a)** Attempt to solve three term quadratic Choose 'outside' region for inequality FT their quadratic inequality in **(b)**, as long as one positive root and one negative root Single correct interval – any correct notation B1M1A0B1 possible if no reason for rejecting – 7	 **BC** Both values of x needed – 7 must be seen and discarded with a reason Condone $5 \leq x < 7.5$

Question			Answer	Marks	AO	Guidance	
3	(a)		**DR**  $\cos\alpha = \pm\frac{1}{3}\sqrt{5}$	M1 A1 [2]	3.1a 1.1	Attempt Pythagoras on correct right-angled triangle Obtain $\pm\frac{1}{3}\sqrt{5}$ oe eg $\pm\sqrt{\frac{5}{9}}$	or $\frac{4}{9}+\cos^2\alpha=1$ $\cos^2\alpha=\frac{5}{9}$ A0 for answer only as **DR**
	(b)		$2\sec^2\beta - 2 - 7\sec\beta + 5 = 0$ $2\sec^2\beta - 7\sec\beta + 3 = 0$ $(\sec\beta - 3)(2\sec\beta - 1) = 0$ $\sec\beta = 3$, $\sec\beta = 0.5$ $\sec\beta \geq 1$ for $0^\circ \leq \beta < 90^\circ$ hence $\sec\beta = 3$	M1 A1 M1 A1 [4]	3.1a 1.1 1.1a 1.1	Attempt to use $\tan^2\beta = \sec^2\beta - 1$ Obtain correct equation Attempt to solve quadratic Obtain $\sec\beta = 3$ only $\sec\beta = 0.5$ must be seen and discarded with a reason	Allow equiv methods involving $\cos\beta$ Allow more general reason such as $\lvert\sec\beta\rvert \geq 1$
4			**DR** $e^x = 3 + 2e^y$ $(3 + 2e^y)^2 - 4e^{2y} = 33$ $9 + 12e^y + 4e^{2y} - 4e^{2y} = 33$ $12e^y = 24$ $e^y = 2$ $y = \ln 2$ $e^x - 4 = 3$ $e^x = 7$ $x = \ln 7$	M1 A1 M1 A1 A1 [5]	3.1a 1.1 1.1a 1.1 2.1	Attempt to eliminate one variable Obtain correct equation in one variable – allow unsimplified Simplify and attempt to solve Obtain $y = \ln 2$ Obtain $x = \ln 7$, using either equation.	or $e^{2x} - 4(0.5e^x - 1.5)^2 = 33$ or $6e^x = 42$ etc

Question			Answer	Marks	AO	Guidance	
5	(a)		$f(x+h)-f(x)=\{(x+h)^2-4(x+h)\}-\{x^2-4x\}$	M1	2.1	Attempt to simplify $f(x+h)-f(x)$	
			$=x^2+2xh+h^2-4x-4h-x^2+4x$				
			$=2xh+h^2-4h$	A1	2.5	Correct expression for $f(x+h)-f(x)$	
			$\dfrac{f(x+h)-f(x)}{h}=\dfrac{2xh+h^2-4h}{h}$	M1	1.1	Attempt $\dfrac{f(x+h)-f(x)}{h}$	
			$=2x+h-4$	A1	2.1	Obtain correct expression	
			$f'(x)=\lim_{h\to 0}(2x+h-4)=2x-4$	A1	2.4	Complete proof by considering limit as $h\to 0$	
				[5]			
	(b)		$y=x^2-4x+c$	B1	3.1a	Correct equation, including c	
			$7=4-8+c$	M1	1.1	Attempt to find c	
			$c=11$				
			$y=x^2-4x+11$	A1	1.1	Obtain correct equation	
				[3]			

Question			Answer	Marks	AO	Guidance	
6	(a)		Geometric sequence, as multiplying by a common ratio each time	**B1** **[1]**	**2.2a**	Identify geometric with reasoning	Allow GP or similar
	(b)		$u_{20} = 500 \times 0.8^{19}$ $= 7.21$	**M1** **A1** **[2]**	**2.1** **1.1**	Attempt u_{20} using ar^{n-1}, with $a = 500$ and $r = 0.8$ Obtain 7.21 or better (7.205759)	**DR** so method must be seen
	(c)		$S_{20} = \dfrac{500(1-0.8^{20})}{1-0.8}$ $= 2471$	**M1** **A1** **[2]**	**2.1** **1.1**	Attempt u_{20} using correct formula, with $a = 500$ and $r = \pm 0.8$ Obtain 2471 or better (2471.17696)	**DR** so method must be seen
	(d)		$\dfrac{u_k}{1-0.8} = 1024$ $u_k = \frac{1024}{5}$ $500 \times 0.8^{k-1} = \frac{1024}{5}$ $0.8^{k-1} = \frac{256}{625}$ $k - 1 = \log_{0.8}(\frac{256}{625}) = 4$ $k = 5$	**M1** **A1** **M1** **M1** **A1** **[5]**	**2.1** **1.1** **2.1** **1.1** **1.1**	Attempt to use correct S_∞ formula, equate to 1024 and attempt u_k Obtain correct value for first term in this sequence Equate $500 \times 0.8^{k-1}$ to their value for u_k and rearrange to $0.8^{k-1} = c$ Correct use of logs to attempt $k - 1$ Obtain $k = 5$	OR attempt $S_\infty - S_{k-1} = 1024$ Could use other notation, OR obtain correct unsimplified equation OR rearrange to $0.8^{k-1} = c$ Allow M1 if using logs to solve $0.8^k = \frac{256}{625}$ **DR** so method must be seen

Question			Answer	Marks	AO	Guidance	
7	(a)		$\frac{dV}{dt} = -kV$	B1	3.3	Set up correct differential equation	Allow k for $-k$
			$-20 = -k \times 500$ so $k = 0.04$	B1	3.3	Correct value for k – may be seen later	Or $k = -0.04$
			$\int -0.04 dt = \int \frac{1}{V} dV$	M1	1.1	Separate variables and attempt integration	
			$-0.04t = \ln V + c$	A1	1.1	Correct integral – could still be in terms of k	Accept no $+ c$ here
			$c = -\ln 500$	M1*	3.4	Use $t = 0$, $V = 500$ to find c	
			$-0.04t = \ln 250 - \ln 500$	M1dep*	3.4	Attempt to find t when $V = 250$	
			$t = 17.3$ hours	A1	3.4	Obtain 17.3 hours, or better (17 hours and 20 minutes)	Units needed 17.3286…
				[7]			
			Alternate method				
			$\frac{dV}{dt} = -kV$	B1	3.3	Set up correct differential equation	Allow k for $-k$
			$-20 = -k \times 500$ so $k = 0.04$	B1	3.3	Correct value for k – may be seen later	Or $k = -0.04$
			$\int_0^T -0.04 dt = \int_{500}^{250} \frac{1}{V} dV$	M1	1.1a	Separate variables and attempt integration of LHS	
				M1	3.4	Use of $t = 0$, $V = 500$	**OR** Use of t limits 0 and T (accept $t = t$)
				M1	3.4	Use of $t = T$, $V = 250$ (accept $t = t$)	Use of V limits 500 and 250 (either way round)
			$-0.04T = -0.693...$	A1	1.1		
			$T = 17$ hours	A1	3.4	Obtain 17.3 hours, or better (17 hours and 20 minutes)	Units needed 17.3286…
				[7]			

Question			Answer	Marks	AO	Guidance	
	(b)		E.g. Assumes that temperature remains constant E.g. Assume that the snowball remains a sphere throughout	B1 [1]	3.5b	Any valid assumption made	
	(c)		Not very realistic as volume never equals 0, so snowball never melts completely	B1 [1]	3.5b	Consider long term prediction	
8	(a)		$(1+2x)^{\frac{1}{2}} = 1 + x$ $+\frac{(\frac{1}{2})(-\frac{1}{2})(2x)^2}{2} + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(2x)^3}{6}$ $= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3$	B1 M1 A1 A1 [4]	1.1 1.1a 1.1 1.1	Obtain correct first two terms Attempt at least one more term Obtain correct third term Obtain correct fourth term	Must be simplified Must be simplified Must be simplified
	(b)		$(1 + 9x^2)^{-1} = 1 - 9x^2$ $(1 - 9x^2)(\ 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3\ )$ $= 1 + x - \frac{19}{2}x^2 - \frac{17}{2}x^3$	B1 M1 A1FT [3]	3.1a 1.1a 1.1	Correct expansion soi Attempt expansion Obtain correct expansion	 FT their **(i)** – must be 4 terms
	(c)		$(1+2x)^{\frac{1}{2}} \Rightarrow \lvert x\rvert < \frac{1}{2}$ $(1+9x^2)^{-1} \Rightarrow \lvert x\rvert < \frac{1}{3}$ hence $\lvert x\rvert < \frac{1}{3}$	M1 A1 [2]	1.1 2.3	At least one correct condition seen Correct conclusion, from both correct conditions	oe oe

Question			Answer	Marks	AO	Guidance	
9	(a)		$f'(x) = -12x(x^2 + a)^{-2}$	M1	3.1a	Attempt differentiation to obtain $kx(x^2 + a)^{-2}$	
				A1	2.1	Obtain fully correct derivative	
			for $x > 0$, $-12x < 0$ and $(x^2 + a)^2 > 0$	M1	2.1	Attempt to show that $f'(x) < 0$	
			negative divided by positive is always negative, hence function is decreasing	E1	2.4	Fully convincing argument	
				[4]			
	(b)		$f''(x) = -12(x^2 + a)^{-2} + 48x^2(x^2 + a)^{-3}$	M1	3.1a	Attempt use of product, or quotient, rule	
				A1	1.1	Obtain correct expression	Allow unsimplified
			$f''(x) = 0$ (and $f'(x) \neq 0$ since $f'(x) = 0$ only when $x = 0$)	B1	1.2	Identify condition for a point of inflection.	Seen or implied Accept non-stationary condition omitted
			$-12(x^2 + a) + 48x^2 = 0$ $36x^2 = 12a$ $x^2 = \frac{a}{3}$	M1	3.1a	Attempt correct process to solve for x	
			$x = \sqrt{\frac{a}{3}}$, $y = \frac{9}{2a}$	A1	1.1	Obtain correct coordinates	A0 if $x = \pm\sqrt{\frac{a}{3}}$
				[5]			

Question			Answer	Marks	AO	Guidance	
10	(a)		At A, $y = 0$ so $-\tan^{-1}(\frac{1}{2}x - \frac{1}{3}\pi) = 0$ $\frac{1}{2}x - \frac{1}{3}\pi = 0$	M1	1.1a	Attempt x-coordinate at A	
			$x = \frac{2}{3}\pi$ so A is $(\frac{2}{3}\pi, 0)$	A1	1.1	Obtain $(\frac{2}{3}\pi, 0)$	Allow decimal equiv 2.094…
			At B, $x = 0$ so $y = 0.808$ so B is $(0, 0.808)$	B1 [3]	1.1	Obtain (0, 0.808), or better	
	(b)		reflection in the x-axis	B1	1.1	Stated at any point	
				M1	1.1a	Translation by $\pm\frac{1}{3}\pi$ and stretch by sf 2 or $\frac{1}{2}$, both in the x direction	Allow informal language for the M1
			translation in the x direction by $\frac{1}{3}\pi$ then stretch in x direction by sf 2	A1 [3]	2.5	Must use ‘factor’ or ‘scale factor’	Allow stretch then translation, as long as details are commensurate with order
	(c)		$0 < 0.808$ $1 > 0.501$	M1	2.1	Substitute $x = 0$ and $x = 1$ into both sides of the equation	Could also rearrange to $f(x) = x + \tan^{-1}(\frac{1}{2}x - \frac{1}{3}\pi)$ and attempt $f(0) = -0.808$ and $f(1) = 0.499$
			change in inequality sign hence $0 <$ root < 1	E1 [2]	2.4	Conclude appropriately	Refer to change in sign
	(d)		eg $x_1 = 0.5$, $x_2 = 0.6730$	B1	1.1a	Correct first iterate for $0 < x < 1$	
			0.6179, 0.6360, 0.6301, 0.6320, 0.6314, 0.6316, 0.6315, 0.6315…	M1	1.1a	Attempt correct iterative process	At least 2 more values
			hence root is 0.632	A1 [3]	1.1	Obtain root as 0.632	Must be 3sf
	(e)		add $y = x$ to diagram in P.A.B. and show first iteration	M1	1.2	Vertical line from x_1 and horizontal line to $y = x$	
			at least 4 more lines to show cobweb	A1 [2]	1.2	ie 2 vertical and 2 horizontal lines	

Question		Answer	Marks	AO	Guidance	
11	(a)	$\dfrac{(x-4)(x+3)+(3x+1)(x+2)}{(x+2)(x-1)(x+3)}$	M1	3.1a	Attempt to use a common denominator	Could be quartic denominator if repeated factor not spotted
			A1	2.1	Obtain correct unsimplified fraction	
		$=\dfrac{x^2-x-12+3x^2+7x+2}{(x+2)(x-1)(x+3)}$	M1	2.1	Expand and simplify numerator	
		$=\dfrac{4x^2+6x-10}{(x+2)(x-1)(x+3)}$				
		$=\dfrac{2(2x+5)(x-1)}{(x+2)(x-1)(x+3)}$	M1	2.1	Attempt to factorise numerator	
		$\dfrac{2(2x+5)}{(x+2)(x+3)}$ **A.G.**	A1	2.1	Obtain given answer, with sufficient detail shown	
			[5]			
	(b)	$\int \mathrm{f}(x)\mathrm{d}x = 2\ln(x^2+5x+6)$	M1	3.1a	Obtain integral of $k\ln(x^2+5x+6)$	no need for modulus signs oe using partial fractions or $2(\ln(x+3)+\ln(x+2))$
			A1	1.1	Obtain correct $2\ln(x^2+5x+6)$	
		$2\ln[(a+4)^2+5(a+4)+6)] - 2\ln(a^2+5a+6)]$	M1	1.1a	Attempt use of limits	Correct order and subtraction
		$2\ln\dfrac{a^2+13a+42}{a^2+5a+6} = 2\ln 3$	M1	3.1a	Equate to 2ln3 and remove logs	Using valid method
		$\dfrac{a^2+13a+42}{a^2+5a+6} = 3$				
		$2a^2+2a-24=0$	A1	1.1	Obtain correct three term quadratic	
		$2(a+4)(a-3)=0$	M1	1.1a	Attempt to solve quadratic	**DR** so method must be seen
		$a=3$	A1	3.2a	Obtain $a = 3$ only	A0 if $a = -4$ also given
			[7]			

Question			Answer	Marks	AO	Guidance	
12	(a)		$\dfrac{\tan 2\theta + \tan\theta}{1-\tan 2\theta \tan\theta}$	B1	2.1	Correct expression	
			$= \dfrac{\dfrac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1-\dfrac{2\tan\theta}{1-\tan^2\theta}\tan\theta}$	B1	2.1	Correct expression in terms of $\tan\theta$	
			$= \dfrac{2\tan\theta + \tan\theta(1-\tan^2\theta)}{(1-\tan^2\theta) - 2\tan^2\theta}$	M1	2.1	Attempt to simplify	As far as clearing fractions
			$= \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$ **AG**	A1	2.1	Complete proof to show given identity convincingly	
				[4]			
	(b)		$3\times\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \tan\theta + k$	M1	3.1a	Equate and attempt to rearrange	
			$9\tan\theta - 3\tan^3\theta = (\tan\theta + k)(1-3\tan^2\theta)$				
			$9\tan\theta - 3\tan^3\theta = \tan\theta - 3\tan^3\theta + k - 3k\tan^2\theta$				
			$3k\tan^2\theta + 8\tan\theta - k = 0$	A1	1.1	Correct 3 term quadratic	
			$b^2 - 4ac = 64 + 12k^2$	A1FT	3.1a	Correct discriminant FT their 3 term quadratic in $\tan\theta$	Could be within quadratic formula
			$k^2 \geq 0$, so $64 + 12k^2 > 0$ so equation will always have two distinct roots	M1	2.2a	Consider sign of correct discriminant and hence number of roots	Discriminant must be correct
			$\tan\theta = c$ will always give one value for θ, which will be between 0° and 90° for $c > 0$ and between 90° and 180° if $c < 0$ so two distinct roots for $\tan\theta$ will always give two values for θ between 0° and 180^0	A1	2.4	Conclude by justifying two values for θ	
				[5]			